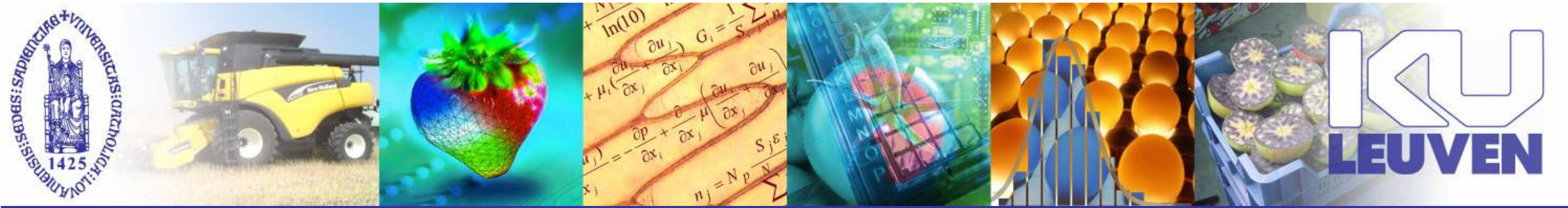

Heat and mass transfer during refrigeration of foods

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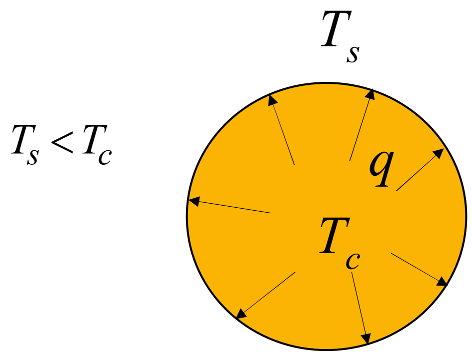
Contents

- Fourier equation and boundary conditions
- Analytical solutions with/without heat generation
- Numerical solution
- Thermophysical properties
- Surface heat transfer coefficient
- Phase change problems
- Mass transfer

Fundamentals of heat transfer

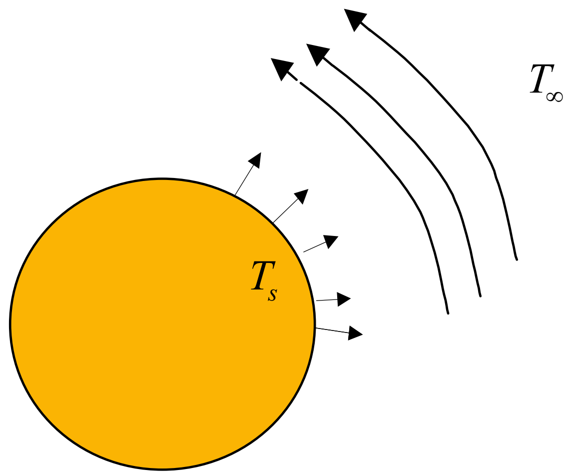
- *Heat transfer :*Energy in transit due to a temperature difference

  *Temperature :*Thermodynamic quantity, measure of movement of molecules (translation, vibration, rotation)
- Modes of heat transfer
  - Conduction heat transfer
    - Heat flow caused by temperature gradient
    - Mechanism
      - vibrating atoms pass their energy to neighbouring atoms
      - vibrations of crystal lattice
      - electrons (metals)

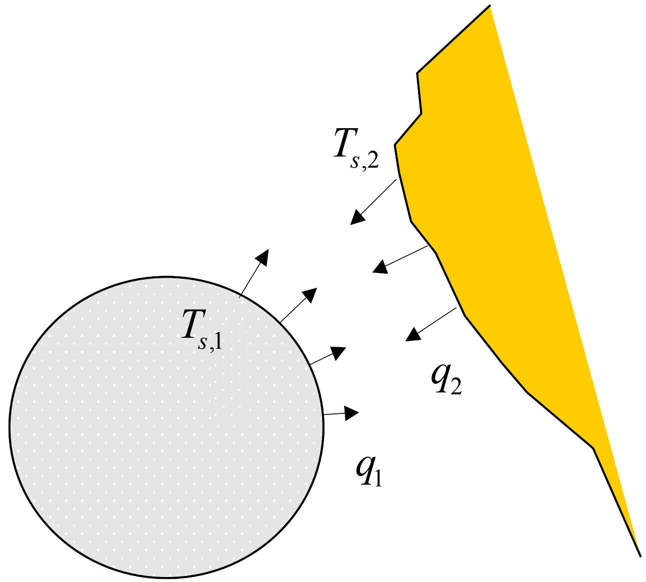


q : heat flux [W/m^2]
T_s : surface temperature [°C]
T_c : centre temperature [°C]

- Convection heat transfer
  - Heat flow caused by physical flow of fluid along surface at temperature
  - Heat transfer to infinitesimal fluid volume by conduction



- Radiation heat transfer
  - All objects radiate
  - Radiation spectrum depends on temperature
  - If two surfaces are at a different temperature, net radiation is different and energy transfer happens
  - No medium required !



Fourier equation

- Fourier's law : heat flux is proportional to temperature gradient

$$q = -k\frac{dT}{dx}$$

with k : thermal conductivity [W/m°C]
T : temperature [°C]

- Fourier's equation

$$\rho c\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + Q$$

with t : time [s]
ρ : density [kg/m^3]
c : heat capacity [J/kg°C]
q : heat flux [W/m^2]
Q : volumetric heat generation [W/m^3] (e.g., respiration)

- 3D $$\rho c\frac{\partial T}{\partial t} = k\nabla^2 T + Q$$

Boundary conditions

- Dirichlet (fixed temperature)

$$T(x,y,z,t)=T_s$$ at Γ, with T_s a known function [°C]

- Neumann (fixed flux)

$$k\frac{\partial T}{\partial n}=-q_s$$ at Γ

with q_s : a known function [W/m^2]

n : outward normal to surface

- Convection

$$k\frac{\partial T}{\partial n}=h(T_\infty - T)$$

with T_∞ : fluid temperature [°C]

h : surface heat transfer coefficient [W/m^2°C]

Radiation boundary condition

$$k\frac{\partial T}{\partial n} = \varepsilon\sigma(T_\infty^4 - T^4)$$

with ε : emissivity [.]

σ : view factor [W/m²°C⁴]

T_∞ : temperature of radiation source [°C]

The same boundary surface may be subject to both radiation and convection

Different parts of the object may be subject to different boundary coditions

Initial condition

$$T(x,y,z,t) = T_0(x,y,z)$$ at t=0, with T_0 a known function

Analytical solution: no heat generation

- For simple geometries, boundary conditions, analytical solutions are available, e.g., from *Carslaw and Jaeger* (1957) "Conduction of heat in solids"
- Dimensionless numbers are introduced to obtain generalised solutions:
  - Dimensionless temperature : $$\theta = \frac{T - T_\infty}{T_0 - T_\infty}$$
  - Fourier number (dimensionless time) : $$Fo = \frac{\alpha t}{L^2}$$
  - Biot number : $$Bi = \frac{hL}{k}$$

  with $\alpha = \frac{k}{\rho c}$: thermal diffusivity [W/m^2]

  L : characteristic dimension of the object [m]
  T_0 : initial temperature [°C]
  T_∞ : ambient temperature [°C]
  h : surface heat transfer coefficient [W/m^2°C]

- Solution is usually an infinite series, containing terms which are roots of transcedental equations, e.g., center temperature for 1D two sided slab with length *2L:*

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty} = \sum_{n=1}^{\infty} A_n \exp\left(-\beta_n^2 Fo\right)$$

with A_n and β_n solutions of the following transcedental equation:

$$A_n = \frac{2\sin\left(\beta_n\right)}{\beta_n + \sin\left(\beta_n\right)\cos\left(\beta_n\right)}$$

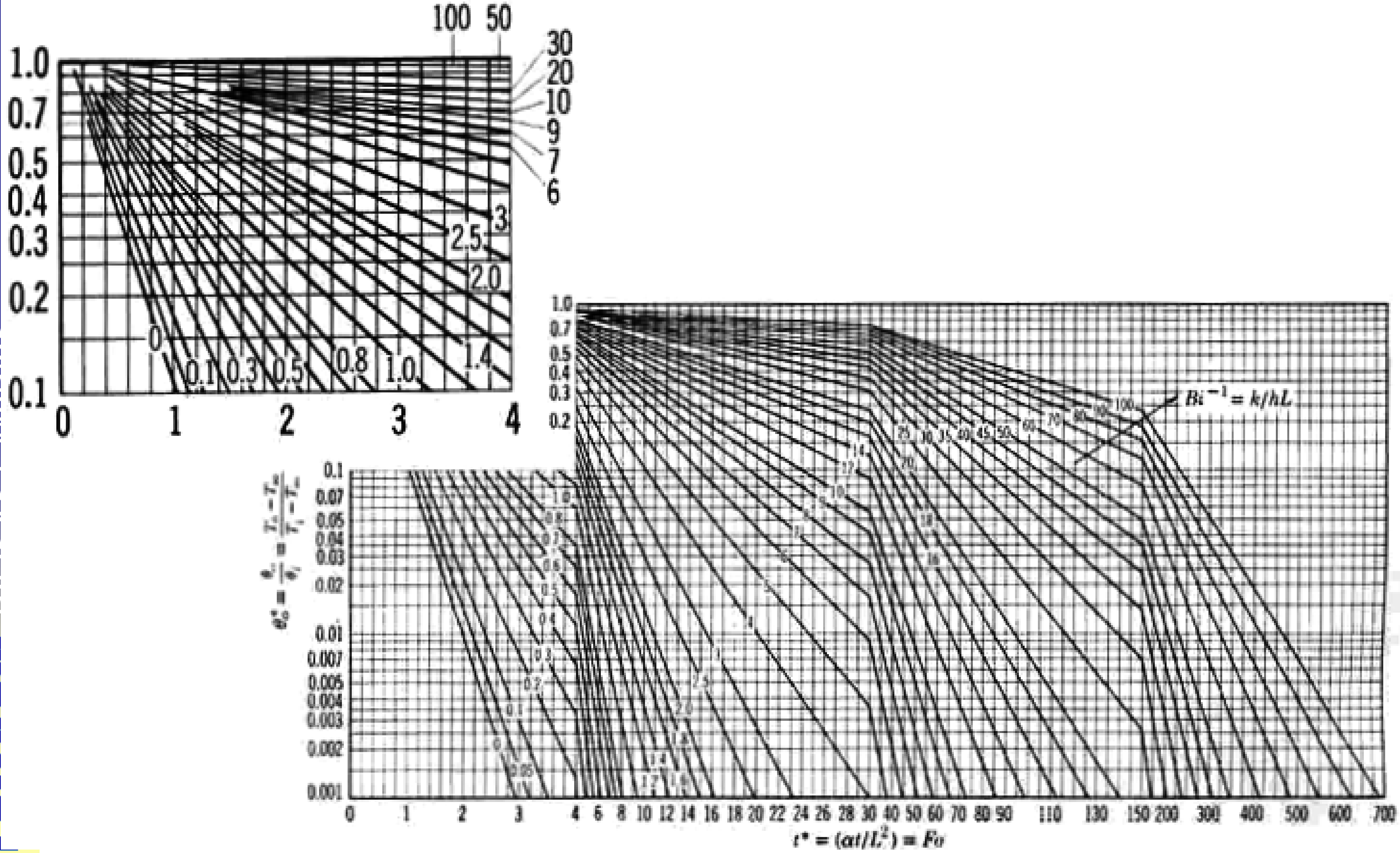
$$Bi = \beta_n \tan\left(\beta_n\right)$$

- Special cases: if Fo > 0.0625 then only 1 term is sufficient

$$\theta_c = A_1 \exp\left(-\beta_1^2 Fo\right)$$

Graphical representation of solution: Heissler charts

E.g., plane wall of thickness 2*L* with convection boundary conditions at both boundaries (Incropera and de Witt, 1983):



Numerical example: Slab ($2L$= 3 cm) filled with water (α=1.4×10^{-7} m^2/s, k=0.6 W/m°C) is initially at 90°C, must be cooled to 10°C with air at a temperature of 2°C. h is equal to 40 W/m^2°C. Calculate the cooling time.

Solution

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{10-2}{90-2} = 0.09$$

$$Bi = \frac{hL}{k} = \frac{40 \times 0.03/2}{0.6} = 1.0$$

From Heissler chart: *Fo*=3.4

$$t = \frac{L^2\,Fo}{\alpha} = \frac{0.015^2 \times 3.4}{1.4 \times 10^{-7}} = 91 \text{ minutes}$$

Analytical solution: constant heat generation

- General solution for center temperature in two-sided slab

$$\theta = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{\text{Po}}{2}\left(1 + \frac{2}{\text{Bi}}\right) + \sum_{n=1}^{\infty}\left(1 - \frac{\text{Po}}{\beta_n^2}\right) \cdot \text{A}_n \exp\left(-\beta_n^2 \text{Fo}\right)$$

with $\text{Po} = \dfrac{QL^2}{k\left(T_0 - T_\infty\right)}$: Pomerantsev number

Q : (constant) heat generation rate

and A_n and β_n solutions of the following transcendental equation:

$$\text{A}_n = \frac{2\sin\left(\beta_n\right)}{\beta_n + \sin\left(\beta_n\right)\cos\left(\beta_n\right)}$$

$$Bi = \beta_n \tan\left(\beta_n\right)$$

- If Fo > 0.0625 then only 1 term is sufficient

$$\theta_c = \frac{\text{Po}}{2}\left(1+\frac{2}{\text{Bi}}\right)+\left(1-\frac{\text{Po}}{\beta_1^2}\right)\cdot A_1 \exp\left(-\beta_1^2 \,\text{Fo}\right)$$

- Steady state temperature

$$\theta_{t\to\infty} \cong \frac{\text{Po}}{2}\left(1+\frac{2}{\text{Bi}}\right)+\underbrace{\left(1-\frac{\text{Po}}{\beta_1^2}\right)\cdot A_1 \exp\left(-\beta_1^2 \,\text{Fo}\right)}_{=0}$$

$$= \frac{\text{Po}}{2}\left(1+\frac{2}{\text{Bi}}\right)$$

- Cooling criterion

$$\theta_{t\to\infty} = \frac{\text{Po}}{2}\left(1+\frac{2}{\text{Bi}}\right) < 1$$

- Example
  - Peppers are packed in boxes with dimensions 0.28×0.48×1.2 m³ and are cooled in an air blast of 2 °C. The initial temperature of the peppers was 25 °C. Other properties are given below:

$$\rho = 200 \text{ kg m}^{-3}$$

$$c_p = 3.984 \text{ kJ kg}^{-1}\,{}^{o}\text{C}^{-1}$$

$$k = 0.136 \text{ W m}^{-1}\,{}^{o}\text{C}^{-1}$$

$$h = 8.7 \text{ W m}^{-2}\,{}^{o}\text{C}^{-1}$$

$$Q_{25^{o}C} = 74 \text{ W m}^{-3}$$

$$\overline{Q} = 26 \text{ W m}^{-3}$$

    - Can the product be cooled under these conditions?
    - What is the lowest temperature that the product can attain?
    - How long do we have to cool to get a center temperature of 5°C?

Solution

Cooling criterion

$$\mathrm{Po} = \frac{QL^2}{k\left(T_i - T_\infty\right)} = \frac{26 \times 0.14^2}{0.136 \times \left(25 - 2\right)} = 0.163$$

$$Bi = \frac{hL}{k} = \frac{8.7 \times 0.14}{0.136} = 8.96$$

$$\frac{\mathrm{Po}}{2}\left(1 + \frac{2}{Bi}\right) = \frac{0.163}{2}\left(1 + \frac{2}{8.96}\right) = 0.1 < 1$$

Steady state temperature

$$\theta_{t\to\infty} = \frac{\mathrm{Po}}{2}\left(1 + \frac{2}{\mathrm{Bi}}\right) = 0.1 \Rightarrow T_{t\to\infty} = 4.3°\mathrm{C}$$

- Final temperature

$$\theta_f = \frac{T_c - T_\infty}{T_i - T_\infty} = \frac{5-2}{25-2} = 0.13$$

$$\mathrm{Bi} = \beta_1 \tan(\beta_1) = 8.96$$

$$\beta_1 = 1.41$$

$$A_1 = \frac{2\sin(\beta_1)}{\beta_1 + \sin(\beta_1)\cos(\beta_1)} = \frac{2\sin(1.41)}{1.41 + \sin(1.41)\cos(1.41)} = 1.26$$

$$\mathrm{Fo} = 1.82 \Rightarrow t = 58h$$

Thermophysical properties

- Literature
  - Anonymous (1993) "*ASHRAE Handbook - Fundamentals (SI edition)",* American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., Atlanta, USA, ISBN 0-910110-97-2
  - Mohsenin, N. (1980) "*Thermal properties of Foods and Agricultural Materials*", Gordon and Breach, London, N.Y., Paris
  - Stroshine R. and Hamann D., "*Physical properties of Agricultural Materials and Food Products*" (1995), course notes, Purdue University, West-Lafayette, USA
  - Databases: http://www.nelfood.com/; http://food.oregonstate.edu/energy/thermal.html

- Calculated based on chemical composition: k, ρ and c are known for major food consituents (water, fat, protein, carbohydrate, fiber, ash)

Heldman, D., 2001, "Prediction models for thermophysical properties of foods", in : Irudayaraj, J., (ed.) Food processing operations modeling Marcel Dekker, New York

- (Constant) Thermal properties of basic food constituents

	ρ_i [kg/m³]	c_i [J/kg°C]	k_i [W/m°C]
Moisture	1000	4200	0.6
Protein	1400	2000	0.18
Fat	925	2000	0.18
Carbohydrates	1540	1550	0.2
Fiber	1450	1850	0.18
Ash	1200	1100	0.33

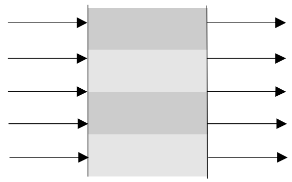
- Thermal properties of ice and water are completely different

	ρ_i [kg/m³]	c_i [J/kg°C]	k_i [W/m°C]
Water	1000	4217	0.569
Ice	920	2040	1.88

- Formulas
  - Heat capacity $c = \sum x_i^m c_i$
  - Density $\rho = \dfrac{1}{\sum x_i^m / \rho_i}$
  - Thermal conductivity

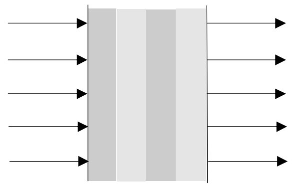
with $x_i^V = x_i^m \dfrac{\rho}{\rho_i}$: volume fraction of component *i*

Parallel model



$$k = \sum x_i^V k_i$$

Serial model

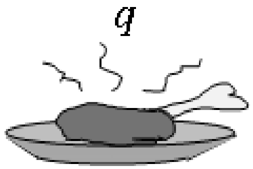
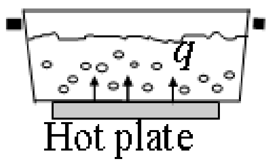
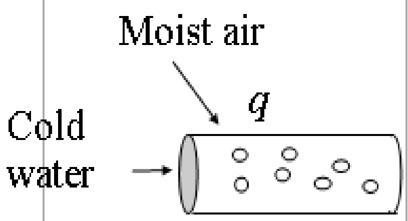


$$\frac{1}{k} = \sum \frac{x_i^V}{k_i}$$

Surface heat transfer coefficient

- Affected by
  - Type of cooling medium
  - Velocity of cooling medium
  - Phase change
  - Shape of food
  - Thermophysical properties of cooling medium

Rules of thumb

Type	Mechanism	Typical values of h ($W/m^2 °C$)	
Forced convection		Gases : Liquids :	25-250 50-20,000
Natural convection		Gases : Liquids :	2-25 50-1000
Boiling			2,500- 100,000
Condensation			2,500- 100,000

- Correlation formulas
  - Symbols

ρ	: density [kg/m^3]
β	: expansion coefficient of convective fluid [m/m]
η	: viscosity of convective fluid [Pa.s]
c	: heat capacity of convective fluid [J/kg°C]
g	: gravity constant (9.8 m^2/s)
k	: thermal conductivity of convective fluid [W/m°C]
L	: characteristic dimension of convection surface [m]
T_s	: surface temperature [°C]
v	: velocity of convective fluid [m/s]

  - Dimensionless numbers
    - Reynolds number $\overline{Re} = \dfrac{\rho v L}{\eta}$
    - Prandtl number $Pr = \dfrac{c\,\eta}{k}$
    - Mean Nusselt number $\overline{Nu} = \dfrac{hL}{k}$
    - Mean Grashoff number $\overline{Gr} = \dfrac{g\beta(T_s - T_\infty)L^3}{(\eta/\rho)^2}$
    - Rayleigh number $Ra = GrPr$

- Correlation equations, general structure:
  - Forced convection: $\overline{Nu} = f(\overline{Re}, Pr)$
  - Free convection: $\overline{Nu} = f(\overline{Gr}, Pr)$
- Thermal properties to be evaluated using film temperature

$$T_{film} = \frac{1}{2}(T_s + T_\infty)$$

- Errors as large as 25%-30%
- Note: for internal flow (flow in tubes, channels, enclosures) consult literature

Correlation equations for external flow, forced convection (Incropera and De Witt, 1990)

Geometry	*Flow regime*	*Correlation*	*Applicability range*	*L*
Horizontal flat plate	Laminar	$\overline{Nu} = 0.664 Re^{1/2} Pr^{1/3}$	$0.6 \le Pr \le 50$	Length of plate
“	Laminar-turbulent	$\overline{Nu} = (0.037 Re^{4/5} - 0.871) Pr^{1/3}$	$0.6 \le Pr \le 60$ $5 \times 10^5 \le Re \le 10^8$ $Re_{x,c} = 5 \times 10^5$ with $Re_{x,c}$ the Reynolds number at the edge of the plate	“
Cylinders in cross flow	Whole range	$\overline{Nu} = 0.3 + \frac{0.62 Re^{1/2} Pr^{1/3}}{\left[1 + (0.4/Pr)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282,000}\right)^{5/8}\right]^{4/5}$	$Re\,Pr > 0.2$	Diameter of cylinder
Sphere	Whole range	$\overline{Nu} = 2 + (0.4 Re^{1/2} + 0.06 Re^{2/3}) Pr^{0.4} \left(\frac{\eta}{\eta_s}\right)^{1/4}$ With η_s the viscosity at the surface temperature All other properties evaluated at T_∞	$0.71 \le Pr \le 380$ $3.5 \le Re \le 7.6 \times 10^4$ $1.0 < \frac{\eta}{\eta_s} < 3.2$	Diameter of sphere

Correlation equations for external flow, free convection (*Incropera and De Witt, 1990*)

Geometry	*Flow regime*	*Correlation*	*Applicability range*	*L*	*Remarks*
Vertical surface	laminar	$\overline{Nu} = \frac{4}{3}\left(\frac{Gr}{4}\right)^{1/4} \frac{0.75Pr^{1/2}}{(0.609 + 1.221Pr^{1/2} + 1.283Pr)^{1/4}}$	$Ra = GrPr < 10^9$, to be evaluated at rear edge of plate	Length of plate	
Vertical plate	laminar+ turbulent	$\overline{Nu} = \left\{0.825 + \frac{0.387Ra^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right\}^2$	Entire range	"	
Horizontal plate	laminar+ turbulent	$\overline{Nu} = 0.54Ra^{1/4}$	$10^4 \le Ra \le 10^7$		Upper surface of heated plate, or lower surface of cooled plate
		$\overline{Nu} = 0.15Ra^{1/3}$	$10^7 \le Ra \le 10^{11}$		
Horizontal plate	laminar+ turbulent	$\overline{Nu} = 0.27Ra^{1/4}$	$10^5 \le Ra \le 10^{10}$		Lower surface of heated plate, or upper surface of cooled plate
Horizontal cylinder	laminar+ turbulent	$\overline{Nu} = \left\{0.60 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right\}^2$	$10^{-5} \le Ra \le 10^{12}$	Dia-meter	
Sphere	laminar+ turbulent	$\overline{Nu} = 2 + \frac{0.589Ra^{1/4}}{\left[1+(0.469/Pr)^{9/16}\right]^{4/9}}$	$Ra \le 10^{11}$, $Pr \ge 0.7$	Dia-meter	

Numerical solution

The Great Wave (after Hokusai, from Kaufman and Smarr, Supercomputing and the Transformation of Science, 1993, Freeman & Co.)

- For most realistic problems (complex shape, nonlinear parameters, time–variable boundary conditions) no analytical solution is available

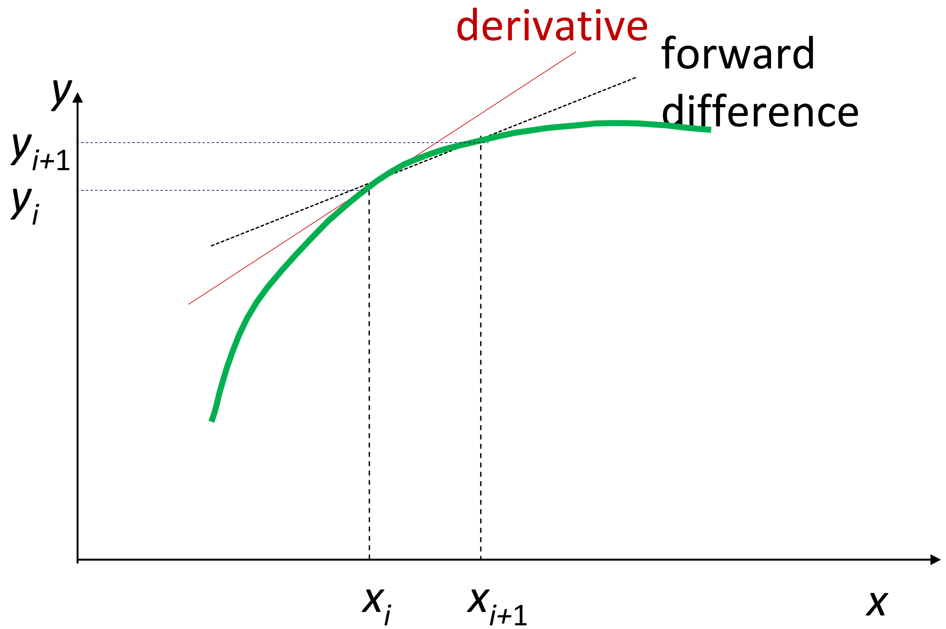
  $\Rightarrow$ discretisation of partial differential equation:

- Procedure
  - Establish mesh (grid) of a discrete number of discretisation points (‘nodes’)
  - Make assumption about behaviour of variable in between grid points, e.g., linear variation
  - Derive for each node differential equation relating the value of the variable in that node to the value of the variable in surrounding nodes
  - Solve resulting system of differential equations in a discrete number of time points

- Accuracy but also necessary computer time in general increases with number of nodes

Finite difference method

- Assume $y = y(x)$
- Definition of derivative: $\left.\frac{d\,y}{d\,x}\right|_{x_i} = \lim_{\Delta x \to 0} \frac{y_{i+1} - y_i}{\Delta x}$
- First (forward) difference $\left.\frac{d\,y}{d\,x}\right|_{x_i} = \frac{y_{i+1} - y_i}{\Delta x} + O(\Delta x)$



- Second order difference $\left.\frac{d^2\,y}{d\,x^2}\right|_{x_i} = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta x^2} + O(\Delta x^2)$

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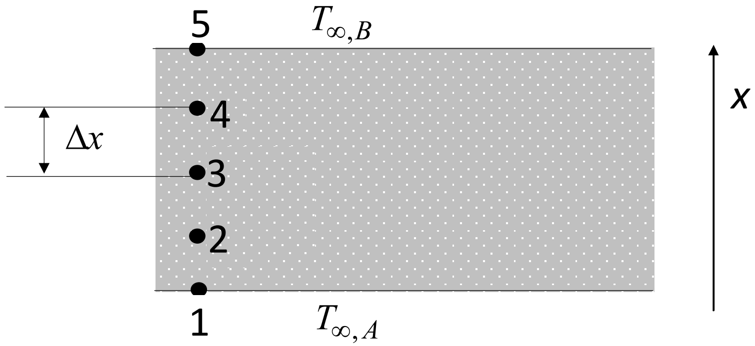
- Fourier equation $$k\frac{\partial^2 T}{\partial x^2} + Q = \rho c \frac{\partial T}{\partial t}$$



  - Node 1,5: $$T_5 = T_{\infty,B}$$ $$T_1 = T_{\infty,A}$$
  - Node 2,3,4:

$$k\frac{T_3 - 2T_2 + T_1}{(\Delta x)^2} + Q_2 = \rho c \frac{dT_2}{dt}$$
$$k\frac{T_4 - 2T_3 + T_2}{(\Delta x)^2} + Q_3 = \rho c \frac{dT_3}{dt}$$
$$k\frac{T_5 - 2T_4 + T_3}{(\Delta x)^2} + Q_4 = \rho c \frac{dT_4}{dt}$$

  - Combine

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho c & 0 & 0 & 0 \\ 0 & 0 & -\rho c & 0 & 0 \\ 0 & 0 & 0 & -\rho c & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt}\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{k}{\Delta x^2} & -\frac{2k}{\Delta x^2} & \frac{k}{\Delta x^2} & 0 & 0 \\ 0 & \frac{k}{\Delta x^2} & -\frac{2k}{\Delta x^2} & \frac{k}{\Delta x^2} & 0 \\ 0 & 0 & \frac{k}{\Delta x^2} & -\frac{2k}{\Delta x^2} & \frac{k}{\Delta x^2} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} T_{\infty,A} \\ -Q_2 \\ -Q_3 \\ -Q_4 \\ T_{\infty,B} \end{bmatrix}$$

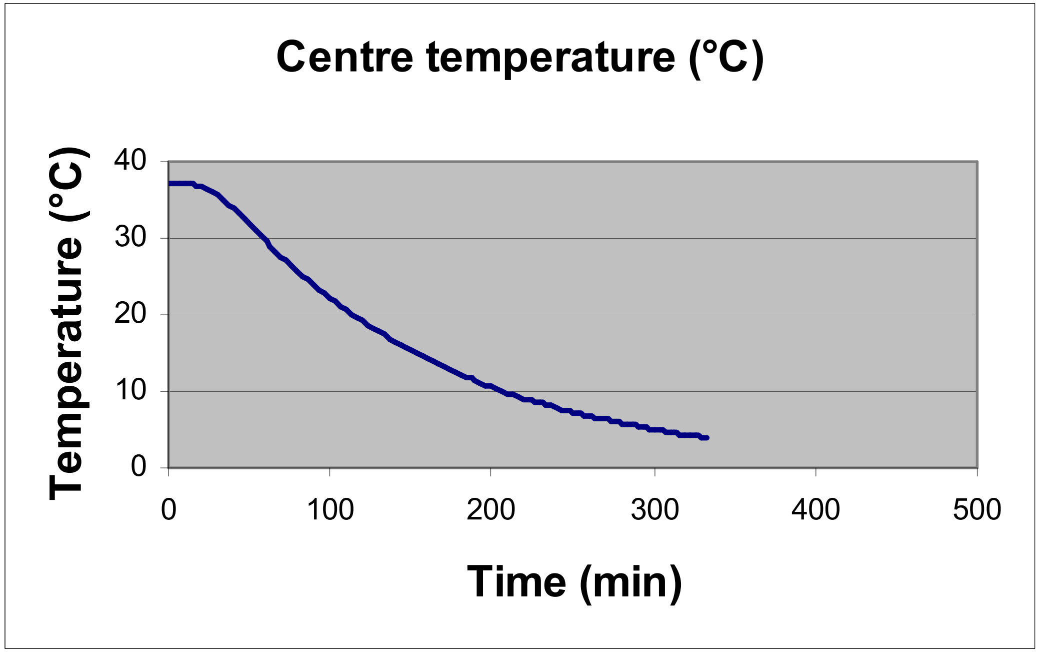
  - Rewrite $$\mathbf{C}\frac{d}{dt}\mathbf{u} + \mathbf{K}\mathbf{u} = \mathbf{f}$$

- Time discretisation (forward difference)

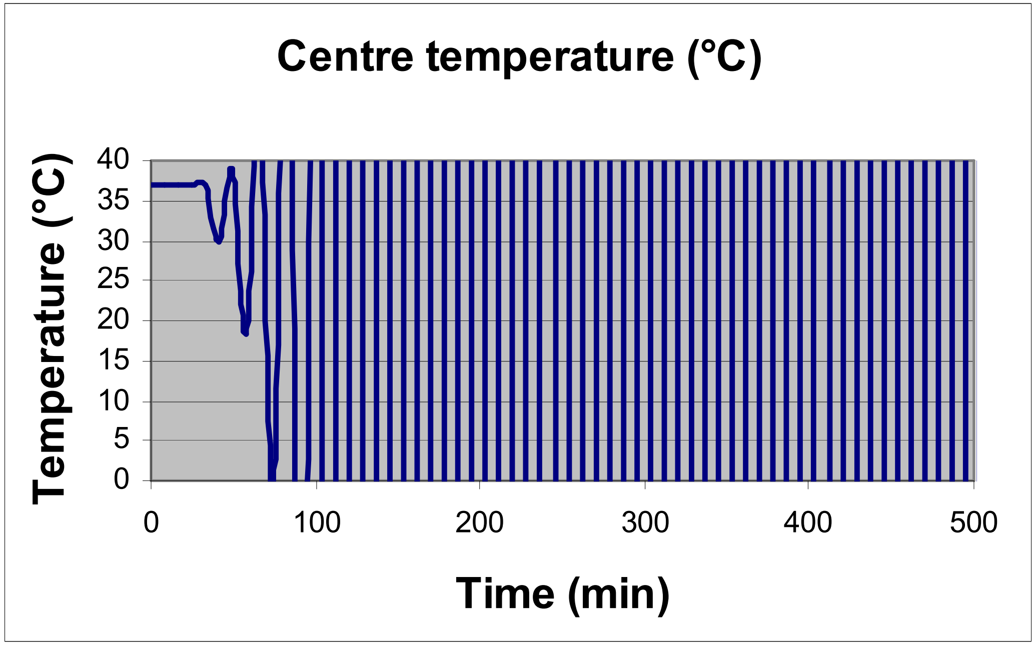
$$\frac{1}{\Delta t}\mathbf{C}\mathbf{u}_{t+\Delta t} + (\mathbf{K} - \frac{1}{\Delta t}\mathbf{C})\mathbf{u}_t = \mathbf{f}_t$$

- Implementation in Excel possible
- Better algorithms available (finite element, finite volume)
- Example (demo)
  - Slab of meat of thickness 10 cm
  - At t=0 T=37°C
  - Temperature at both bottom and top set to 0°C
  - k=0.5 W/m°C, ρ=1000 kg/m^3 and c=4000 J/kg°C

$\Delta t = 200$ s

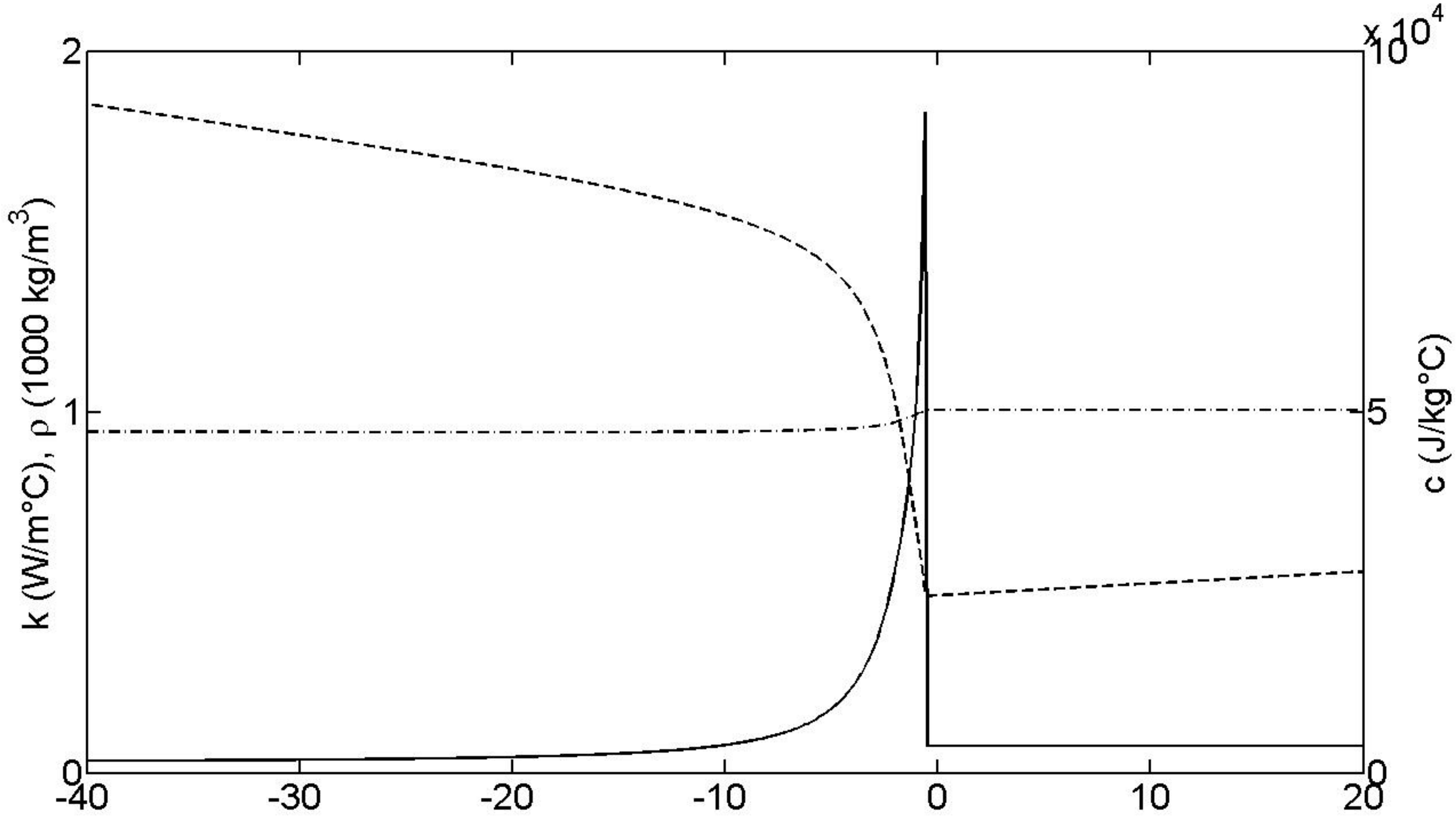


$\Delta t = 500$ s



Phase change problems

- Phase change problems can be considered as nonlinear heat conduction problems with thermophysical properties which highly depend on temperature

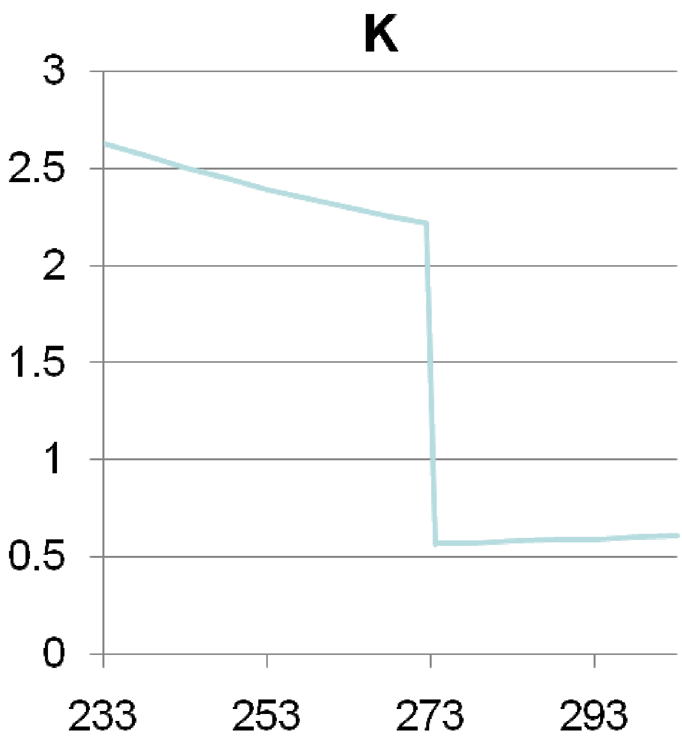


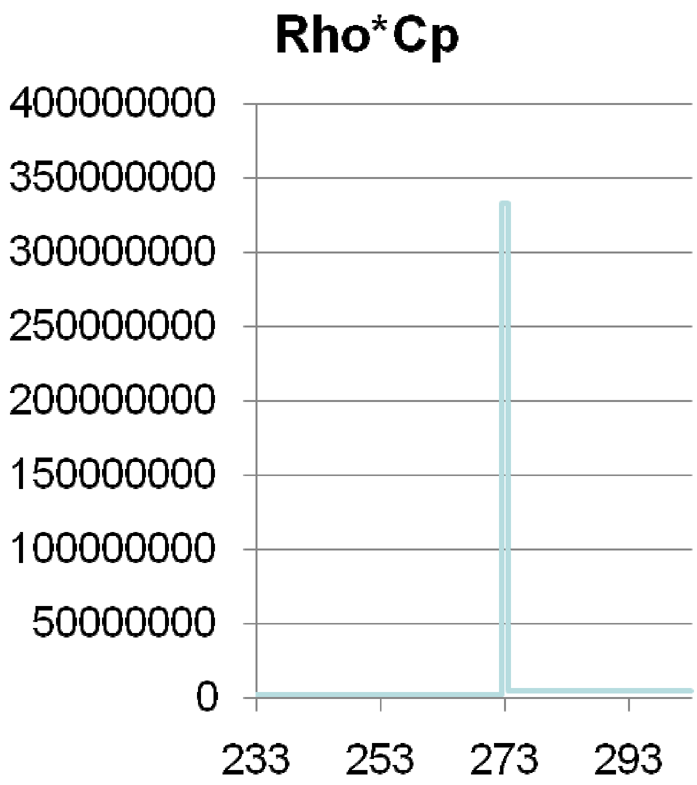
Thermophysical properties of tylose (meat model)

- Implementation with finite difference method needs special care
  - Small time steps
  - Implicit finite difference methods (solution of nonlinear systems required)
  - Enthalpy methods

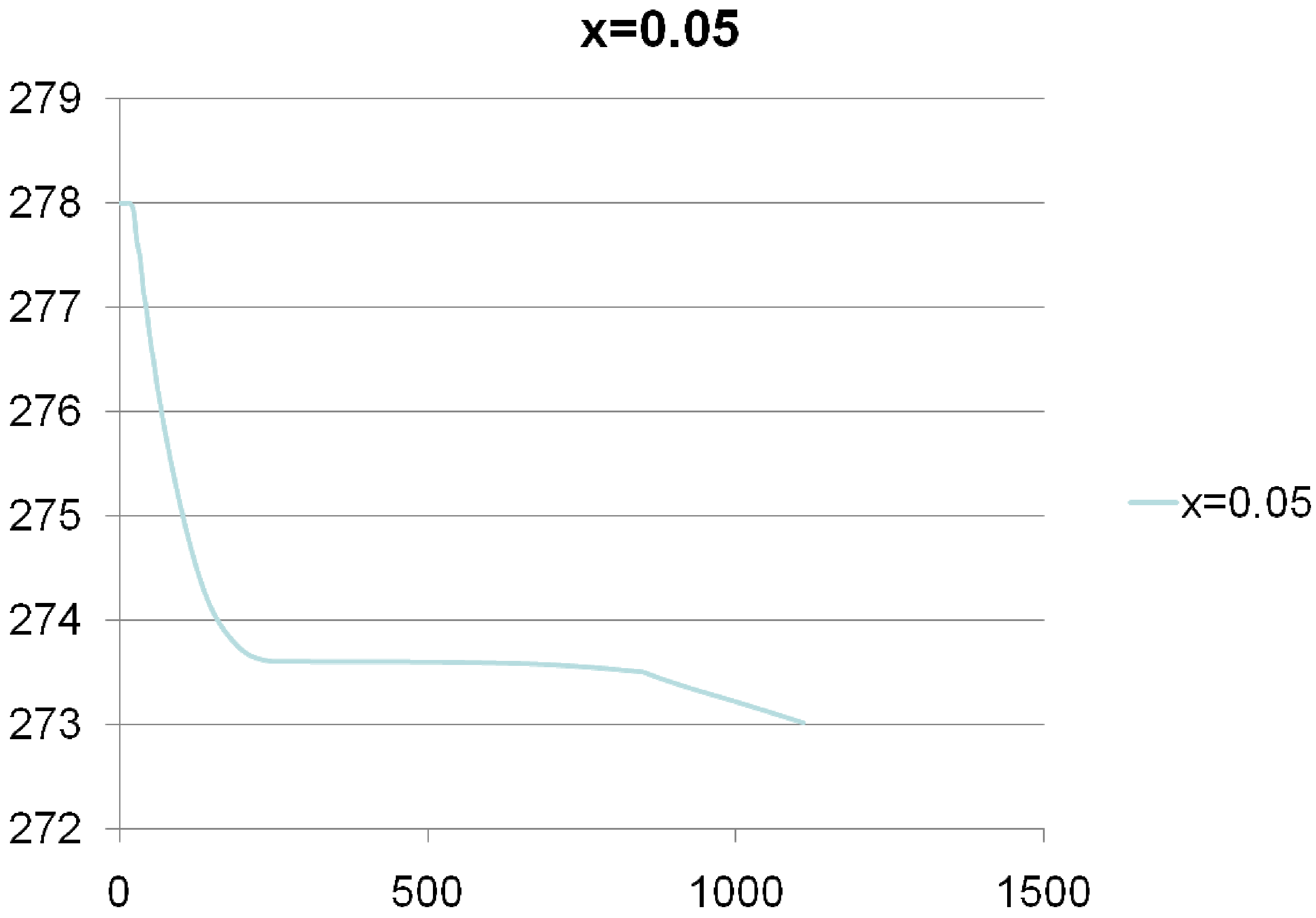
- Example (demo):
  - Freezing of slab
  - L=0.1 m
  - T_0=5 °C
  - T_∞: -3°C
  - Fine mesh and small time steps required !

- Themophysical properties
  - Model latent heat as apparent c_p





Typical temperature profile in center



Mass transfer

- Similar physics, e.g., Fick's equation

$$\frac{\partial c}{\partial t} = D\nabla^2 c$$

With D : diffusion coefficient of moisture in food [m^2/s]

c : moisture concentration [kg/m^3]

- Convection boundary condition

$$-D\frac{\partial c}{\partial x} = h_m(K^* c - c_\infty)$$

  - With h_m : surface mass transfer coefficient [m/s]

    K^* : partition coefficient

    c_∞ : moisture concentration in air [kg/m^3]

- Lewis analogy
  - Similarity between boundary layers for heat and moisture transfer

Heat transfer

$$\overline{\mathrm{Nu}} = \frac{hL}{k} = f\left(\overline{\mathrm{Re}}, \mathrm{Pr}\right)$$

$$\mathrm{Pr} = \frac{\mu c_p}{k} = \frac{\mu / \rho}{k / \rho c_p} = \frac{\upsilon}{\alpha}$$

Moisture transfer

$$\overline{\mathrm{Sh}} = \frac{h_m L}{D_{AB}} = f\left(\overline{\mathrm{Re}}, \mathrm{Sc}\right)$$

$$\mathrm{Sc} = \frac{\mu / \rho}{D} = \frac{\upsilon}{D}$$

Existing correlation formula’s for heat transfer can be used !

Take home messages

- Heat conduction in foods is described by Fourier's equation
- Analytical solutions exist for Fourier's equation with and without heat generation which depends linearly on temperature
- Thermophysical properties can be calculated based on chemical composition
- Correlation equations exist to calculate the surface heat transfer coefficient
- More complex problems, including phase change problems, can be solved conveniently using the finite difference method
- Physics of moisture transport is very similar to that of heat transfer
- Due to the Lewis analogy, the correlation equations for the surface heat transfer can be used to calculate the surface mass transfer coefficient

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